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TECHNICAL NOTES

One-dimensional conduction–radiation heat transfer between parallel surfaces subject to convective boundary conditions

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NOMENCLATURE

a	absorption coefficient [m^{-1}]
B	Biot number, $h[k(a+\sigma_s)]^{-1}$
E	defined in equation (6)
h	heat transfer coefficient [$W\ m^{-2}\ K^{-1}$]
I	moment of radiation intensity
k	thermal conductivity [$W\ m^{-1}\ K^{-1}$]
L	distance between the two surfaces [m]
N	radiation–conduction parameter, $k(a+\sigma_s)(4\sigma T^3_{\infty,1})^{-1}$
q	non-dimensionalized heat flux
R	defined in equation (6)
S	$sL(\sigma T^4_{\infty,1})^{-1}$
s	volumetric heat generation [$W\ m^{-3}$]
z	coordinate axis [m].

Greek symbols

ρ	reflectivity
σ	Stefan–Boltzmann constant [$W\ m^{-2}\ K^{-4}$]
σ_s	single scattering coefficient [m^{-1}]
τ	optical depth
ω	single scattering albedo.

Subscript

1, 2	surface 1 or 2
∞	ambient
b	black body
c	conductive
d	diffuse
i	surface i
r	radiative
s	specular
t	total.

Superscript

$'$	τ-derivative.

1. INTRODUCTION

ANALYSIS of the combined conduction and radiation problem has been receiving substantial attention and a thorough review of recent studies is given in ref. [1]. The trend has been that of improvement in the accuracy of the solutions while including more complicated physical systems.

One method of solution that has been applied to multi-dimensional problems involving combined modes and absorbing–emitting–scattering media is the P_3 approximation [2–4]. This method is more accurate than the P_1 approximation which is the same as the two-flux model. Except for an optically thin media, this method has been successful in predicting the temperature and heat transfer in a variety of geometries [3, 4]. However, as is the case for most of the studies to date, the problems that have been considered have the surface temperatures specified. When the boundary conditions are of the general convective type, and when the intensity of the conductive heat transfer at different boundaries is significantly different, the resulting asymmetry will have interesting features not present when the surface temperatures are prescribed.

In this study a small extension is made to the application of the P_3 approximation method [3, 4], by considering thermal boundary conditions to be of the general, convective type. The problem considered is that of one-dimensional and steady-state, radiative–conductive heat transfer in a gray and emitting–absorbing–scattering medium with uniform heat generation. The medium is confined between parallel, gray surfaces which can reflect diffusely and specularly. By allowing different magnitudes of convective cooling intensity at the two surfaces, the problem will become asymmetric even if the radiative properties of the surfaces are identical. Then the effects of various parameters of the problem on the temperature and heat flux distributions are examined.

2. GOVERNING EQUATIONS

In the P_3 approximation [2–4], determination of the radiation intensity begins by expressing this quantity in terms of spherical harmonics. The coefficients in these series are determined in terms of the moment of the radiation intensity. These moments, which then need to be determined, are governed by the equations which are obtained by multiplying the equation of radiative transfer by different powers of the direction cosine and integrating the results over the complete range of solid angle. The unknown temperature distribution and the radiative heat flux can then be determined from these moments. These equations (except for the convective boundary conditions) are developed in refs. [3, 4], but the final forms are repeated below for completeness.

$$I'_1-(1-\omega)(4T^4-I_0) = 0 \tag{1}$$

$$I'_{11}+I_1 = 0 \tag{2}$$

$$I'_{111}+I_{11}-3^{-1}(1-\omega)(4T^4-I_0)-3^{-1}I_0 = 0 \tag{3}$$

$$I'_{11}+10^{-1}I'_0+(7/6)I_{111} = 0 \tag{4}$$

$$I'_1-4NT''-\tau_L^{-1}S = 0 \tag{5}$$

$$3I_0\pm 16(1+2E_i)I_1+15I_{11}-32T^4 = 0 \tag{6a}$$

$$-(2+5R_i)I_0\pm 16R_iI_1+15(2+R_i)I_{11} \pm 32(1+2E_i-R_i)I_{111}-32T^4 = 0, \quad i = 1,2 \tag{6b}$$

where

$$E_i = (\rho_{s,i}+\rho_{d,i})(1-\rho_{s,i}-\rho_{d,i})^{-1}, \quad R_i = \rho_{d,i}(1-\rho_{s,i}-\rho_{d,i})^{-1}, \quad i = 1,2$$

and the positive signs are for surface 1.

The convective boundary conditions for the case of perfectly conducting walls are

$$\pm T\mp T_{\infty,i} = B_i^{-1}T'+(4B_iN)^{-1}I'_{11}, \quad i = 1,2 \tag{6c}$$

where the positive sign for T is for surface one. The coefficient $4B_iN$ which is equal to $h_i(\sigma T^3_{\infty,1})^{-1}$ can be called the convective–radiative boundary condition parameter. The two special cases of prescribed surface temperature and thermal insulation boundary conditions are for Biot numbers, B, with magnitudes of infinity and zero, respectively.

Equations (1)–(5) are reduced to three second-order ordinary differential equations with I_0, I_{11} and T as unknowns. Equations (6a)–(6c) supply the six boundary conditions required for a complete description of the problem.

The governing equations are solved numerically by using finite-difference approximations. The convergence criterion was that the overall energy balance be satisfied to less than $10^{-6}\%$ of the integrated heat generation.

3. RESULTS AND DISCUSSION

The results of the numerical integration will be shown graphically and are for $B_1 = \infty$ and $T_{\infty,1} = T_{\infty,2}$. Therefore, surface 1 has a prescribed temperature of $T_{\infty,1}$. Furthermore, since

$$q_t = q_r + q_c = I_1 - 4NT'$$

and since we also have

$$\int_0^1 q_t' \,\mathrm{d}z = S$$

then by normalizing the heat flux with respect to S, the non-dimensionalized volumetric heat generation need not be used as a variable. The final parameters of the problem are: B_2, N, τ_L, ρ's and ω. When B_2 is not equal to B_1, the system will not have any symmetry axes, regardless of the magnitudes of the surface properties. The effects of the above mentioned parameters on this asymmetry are the phenomena of interest here.

3.1. *Effect of Biot number*

Figure 1 shows the effect of B_2 on the radiative heat flux for $N = 0.1$, $\tau_L = 1$, $\rho_{d,1} = \rho_{s,1} = \rho_{d,2} = \rho_{s,2} = 0$ and $\omega = 0$. Note that as B_2 decreases the radiative heat flux at surface 2 decreases and its sign gradually changes in order to compensate for the conductive heat flux. This energy in turn has to be absorbed by surface 1. The results show that for the set of parameters used, the significant effect on heat transfer is when $1 < B_2 < 10^2$.

3.2. *Effect of scattering albedo*

In order to illustrate the effect of the magnitude of the scattering albedo on heat transfer, the other parameters are kept the same as before except for B_2, which is given a value of

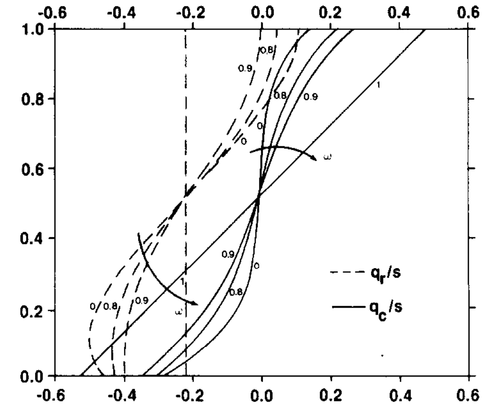

FIG. 2. The effect of the magnitude of the scattering albedo on the distributions of the conductive and radiative heat fluxes. The results are for $B_2 = 10$. The rest of the parameters are the same as those for Fig. 1.

10 in order to emphasize asymmetry. In general, the effect of scattering is to reduce the magnitude of the radiative heat flux. When there is internal heat generation, conductive heat flux will then compensate for this effect. This trend is shown in Fig. 2. The results show that the effect of scattering becomes important only when ω is significantly large. Specifically, the effect of scattering is most pronounced when near complete scattering is approached, i.e. $\omega > 0.8$.

3.3. *Effect of conduction–radiation parameter*

As the results for the distribution of conductive heat flux indicate (for the specific cases considered), the conductive mode is only important in regions adjacent to the surfaces. As N increases, these regions will expand further towards the centerline. Figure 3 shows such a trend for $\tau_L = 1$, $B_2 = 10$, $\omega = 0$ and black surfaces. The heat transfer is most sensitive to $0.1 < N < 1$. The results for $N < 0.1$ are similar to those for $N = 0.1$ and are not shown in this figure. For $N = 0.1$ most of the heat is transferred by radiation except near the surfaces, and this limited contribution from conduction becomes even smaller as N decreases.

3.4. *Effect of surface reflectivities*

As the reflectivity of surface 2 increases, the amount of heat removed from this surface by radiation decreases. Similar results occur at surface 1 as the reflectivity of this surface increases, but the relative magnitude of these reductions depends on the Biot numbers. Figure 4 shows the results for N

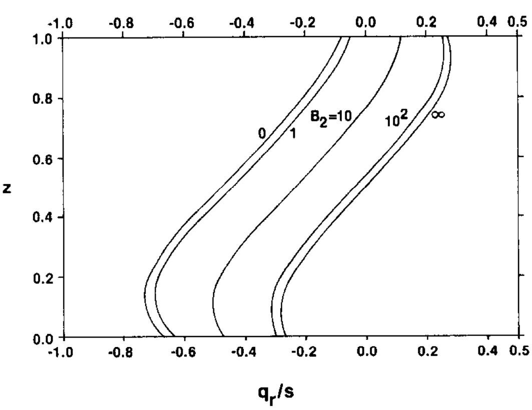

FIG. 1. The effect of the value of the Biot number of surface 2 on the radiative heat flux. The results are for $B_1 = \infty$, $T_{\infty,1} = T_{\infty,2}$, $N = 0.1$, $\tau_L = 1$, $\omega = 0$ and black surfaces.

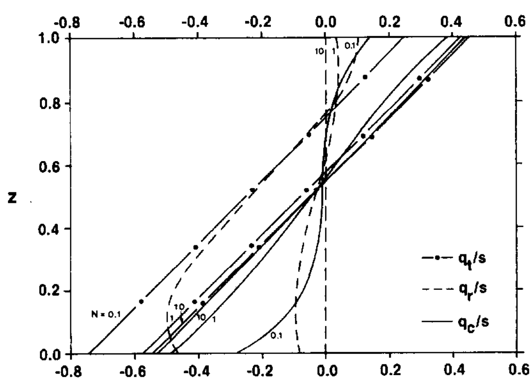

FIG. 3. The effect of the magnitude of the conduction–radiation parameter on the distribution of the heat fluxes. The results are $B_2 = 10$. The rest of the parameters are the same as those for Fig. 1.

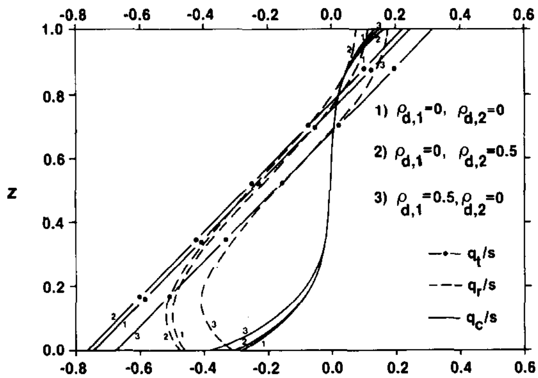

FIG. 4. The effect of the magnitude of the surface reflectivities on the distributions of the heat fluxes. The results are for $B_2 = 10$. The rest of the parameters are the same as those for Fig. 1.

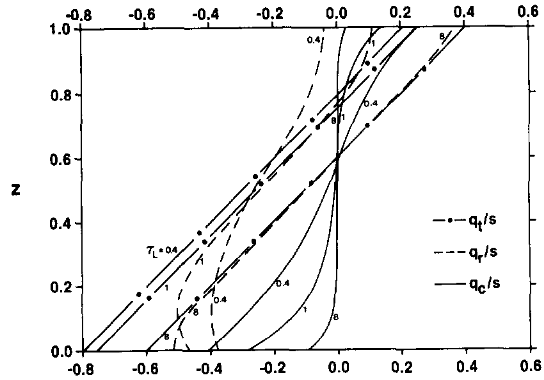

FIG. 5. The variation of the distributions of the heat fluxes with respect to the optical-thickness. The results are for $B_2 = 10$. The rest of the parameters are the same as those for Fig. 1.

$= 0.1, \tau_L = 1, \omega = 0$ and $B_2 = 10$. In case 2 only the reflectivity of surface two is increased to 0.5, while in case 3 this change is made at surface 1. However, since for this set of Biot numbers the radiative heat transfer from surface 1 is larger, the change in the magnitude of the radiative heat flux is also larger for this surface. This is also evident from the change in the distribution of the total heat flux which is less pronounced when the reflectivity of surface 2 is increased.

The results obtained by splitting the magnitude of the reflectivity between diffuse and specular components indicate that the overall heat transfer is not altered by this splitting (not shown graphically). This is true regardless of which surface this splitting is specified for.

3.5. *Effect of optical thickness*

As the optical thickness increases, i.e. the ability for the distant exchange of energy decreases, the medium temperature increases which in turn increases the radiative heat transfer from the medium. Figure 5 shows the heat flux distributions. The results show that as τ_L increases the total heat transferred from surface 2 increases. This increase is almost entirely due to the radiative transport. It should be noted that although temperature gradients near both surfaces increase with an increase in τ_L, the conductive heat flux, which is the derivative with respect to τ, decreases with an increase in τ_L. Therefore, as τ_L increases the total heat transfer from surface 2 increases while the conduction contribution to the heat transfer decreases.

4. CONCLUSIONS

Based on the results obtained, the following conclusions can be made:

(a) For the case where one surface is kept at a prescribed temperature, the effect of the Biot number of the second surface is only significant when the magnitude of this parameter is between 1 and 100. The effect of this Biot number is mostly noticeable on the distribution of the radiative heat flux, i.e. the conductive heat flux is less sensitive to changes in this Biot number.

(b) The effect of increasing the scattering albedo is to reduce the radiative heat flux, but it is not very significant until this parameter exceeds a value of 0.8.

(c) The effect of the conduction–radiation parameter is most significant when this parameter has a value between 0.1 and 1.

(d) When the reflectivity of the surfaces is changed, the effect is more pronounced on the surface with the larger heat transfer rate. For the same total reflectivity, the manner in which this is split between the diffuse and specular component does not alter the overall heat transfer results.

(e) As the optical thickness increases, more heat is transferred from surface 2 and the contribution of the conductive heat flux to the overall heat transfer decreases.

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